

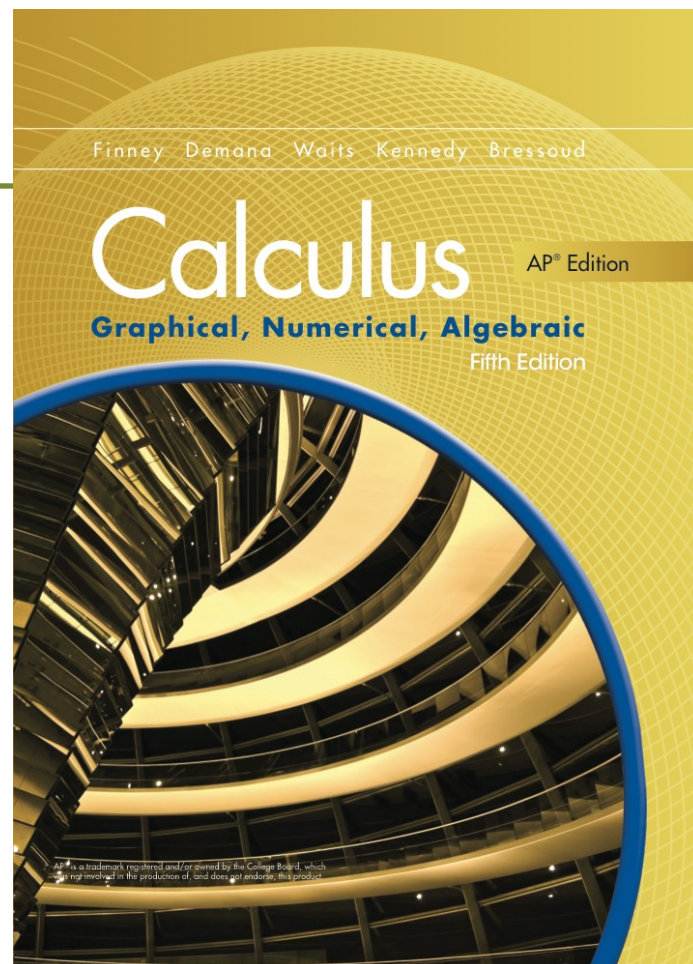
Chapter 3

Derivatives



Section 3.4

Velocity and Other Rates of Change



Quick Review

In Exercises 1 – 10, answer the questions about the graph of the quadratic function $y = -16x^2 + 160x - 256$ by analyzing the equation algebraically. Then support your answers graphically.

1. Does the graph open upward or downward?
2. What is the y -intercept?
3. What are the x -intercepts?
4. What is the range of the function?
5. What point is the vertex of the parabola?

Quick Review

6. At what x -values does $f'(x) = 80$?
7. For what x -value does $\frac{dy}{dx} = 100$?
8. On what interval is $\frac{dy}{dx} > 0$?
9. Find $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$.
10. Find $\frac{d^2 y}{dx^2}$ at $x=7$.

Quick Review Solutions

In Exercises 1 – 10, answer the questions about the graph of the quadratic function $y = -16x^2 + 160x - 256$ by analyzing the equation algebraically. Then support your answers graphically.

1. Does the graph open upward or downward? **Downward**
2. What is the y -intercept? **y - intercept = - 256**
3. What are the x -intercepts? **x - intercepts = 2, 8**
4. What is the range of the function? **$(-\infty, 144]$**
5. What point is the vertex of the parabola? **$(5, 144)$**

Quick Review Solutions

6. At what x -values does $f(x) = 80$? $x = 3, 7$

7. For what x -value does $\frac{dy}{dx} = 100$? $x = \frac{15}{8}$

8. On what interval is $\frac{dy}{dx} > 0$? $(-\infty, 5)$

9. Find $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$. 64

10. Find $\frac{d^2 y}{dx^2}$ at $x = 7$. -32

What you'll learn about

- Instantaneous rates of change
- Motion on a line
- Acceleration as the second derivative
- Modeling vertical motion and particle motion
- The derivative as a measure of sensitivity to change
- Marginal cost and marginal revenue

... and why

Derivatives give the rates at which things change in the world.

Instantaneous Rates of Change

The instantaneous rate of change of f with respect to x at a is the derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

When we say rate of change, we mean instantaneous rate of change.

Example Instantaneous Rates of Change

If the area of a square as a function of the radius is $A = \pi r^2$,

- (a) Find the rate of change of the area A with respect to the radius r .
- (b) Evaluate the rate of change of A when $r=4$.

(a) The rate of change is the derivative $\frac{dA}{dr} = 2\pi r$

(b) The rate of change when $r=4$ is $2\pi(4) = 8\pi$

Motion Along a Line

Suppose that an object is moving along a coordinate line so that we know its position s on that line as a function of time t :

$$s = f(t)$$

The **displacement** of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$.

The **average velocity** of the object over that time interval is

$$V_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Instantaneous Velocity

The (instantaneous) velocity is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}.$$

Speed

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left| \frac{ds}{dt} \right|$$

Acceleration

Acceleration is the derivative of velocity with respect to time.

If a body's velocity at time t is $v(t) = \frac{ds}{dt}$ then the body's

acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Free-fall Constants (Earth)

English Units	$g=32\frac{\text{ft}}{\text{sec}^2},$	$s=\frac{1}{2}(32)t^2=16t^2$	(s in feet)
Metric Units	$g=9.8\frac{\text{m}}{\text{sec}^2},$	$s=\frac{1}{2}(9.8)t^2=4.9t^2$	(t in meters)

Example Finding Velocity

A projectile is shot upward from the surface of the earth and reaches a height of $s = -4.9t^2 + 120t$ meters after t seconds.

Find the velocity after 5 seconds.

$$v = \frac{ds}{dt} = -9.8t + 120$$

when $t=5$, $v = -9.8(5) + 120 = 71$ meters per second

Sensitivity to Change

When a small change in x produces a large change in the value of a function $f(x)$, we say that the function is relatively sensitive to changes in x . The derivative $f'(x)$ is a measure of this sensitivity.

Derivatives in Economics

Economists have a specialized vocabulary for rates of change and derivatives. They call them **marginals**. In a manufacturing operation, the **cost of production** $c(x)$ is a function of x , the number of units produced. The **marginal cost of production** is the rate of change of cost with respect to the level of production, so it is $\frac{dc}{dx}$.

Sometimes the marginal cost of production is loosely defined to be the extra cost of producing one more unit.

Example Derivatives in Economics

Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$.

Find the marginal cost of producing 100 washing machines.

The marginal cost is $c'(x) = \frac{d}{dx}(2000 + 100x - 0.1x^2) = 100 - 0.2x$

The marginal cost of producing 100 machines is $100 - 0.2(100) = \$80$